

# Probabilistic Assessment of Roadway and Railway Viaducts

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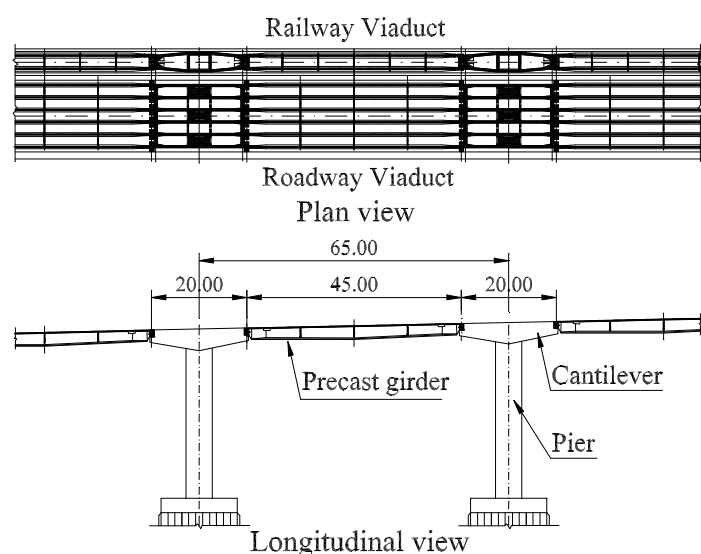
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## Summary

The paper presents a probabilistic procedure for evaluation of the structural safety of roadway and railway viaducts and its application to the viaducts of two major river crossings located over the Paraná river in Argentina. The proposed procedure, that leads to the reliability indexes of the bridges as a measure of their structural safety, was applied for evaluation of the most critical sections and loading conditions of both roadway and railway viaducts, and its results compared with those of a deterministic evaluation. The analyses performed take into account the real loading conditions as described by actual measured data of roadway and railway traffic, as well as the results of inspections and tests performed on the present properties of the materials and parameters of structural performance obtained through dynamic tests.

**Keywords:** assessment; existing structures; structural safety; index of reliability.



**Figure 1.** Typical span of the roadway and railway viaduct of the Zárate-Brazo Largo Complex

## 1. Introduction

The Zárate-Brazo Largo roadway and railway complex consists of two cable stayed bridges and the corresponding access road and rail viaducts crossing two branches of the Paraná River between the city of Zárate and the township of Brazo Largo in the border of the Provinces of Buenos Aires and Entre Ríos in Argentina.

The roadway part of the complex provides two lanes in each direction while the railway part has only one medium size track serving both directions of traffic. The total length of roadway viaducts of the two crossings is 6415 m and that of railway viaducts is 9888 m.

The probabilistic structural assessment of the viaducts described in this paper is part of the results of a comprehensive study undertaken by a consortium conformed by COWI Consulting Engineers of Denmark, The Politechnical University of Catalunya of Spain and the consulting engineering firm SETEC SRL of Córdoba, Argentina. This work was performed under contract with the bridge owner, the National Highway Administration of Argentina, with the objectives of structural evaluation, repair design and maintenance of the both cable stayed bridges and the viaducts.

## 2. Characteristics of the viaducts

### 2.1 Roadway viaducts

The roadway viaducts consist of simple spans of 45 m length supported by symmetrical cantilevers of the piers through halving joints. The deck is made of 6 variable depth pre-cast post-tensioned girders of “I” cross section connected by 5 transverse pre-cast post-tensioned concrete beams and a cast-in-place reinforced concrete slab filling the gaps between the compression flanges of the main girders.

Each pier consists of 3 columns of hollow rectangular cross section with a common single set of symmetrical cantilevers spanning 10 m on both sides that provide support for the simple spans, thus producing typical 65 m spans between pier axes. The cantilevers are cast-in-place reinforced concrete multiple-cell box girders with 6 variable depth webs connected at the top by the deck slab and at the bottom by a slab of variable thickness. The ends of the cantilevers which provide support for the simple spans through a halving joint are connected in the transverse direction by a concrete beam. The cantilevers provide transverse frame action for the three columns that make up each pier. Figure 1 shows a typical span of the roadway viaduct.

### 2.2 Railway viaducts

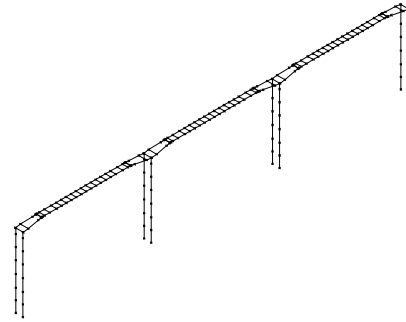
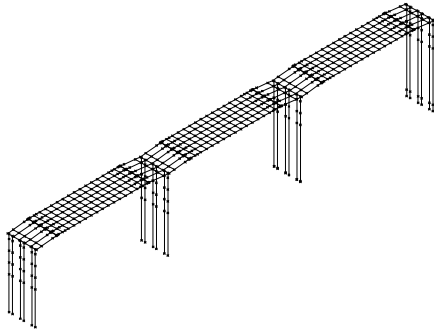
The railway viaducts are also made of simple spans of 45 m length supported by symmetrical cantilevers of the piers by halving joints. The deck is made of 2 variable depth pre-cast post-tensioned girders of “I” cross section connected by 7 transverse pre-cast post-tensioned concrete beams and a cast-in-place reinforced concrete slab of 0.25 m thickness filling the gap between the compression flanges of the main girders to supports the ballast and tracks.

The piers are made of a single hollow rectangular cross section column with two different configurations: i) Piers with symmetrical cantilevers of 10 m on both sides of the column axis for the higher section of the viaducts, and ii) Piers without cantilevers for the lower sections. The cantilevers consist of a cast-in-place single-cell box girder with two webs connected both at the top and bottom by concrete slabs. Figure 1 shows a typical span of the railway viaduct with cantilevers.

## 3. Models of analysis

### 3.1 Numerical model of the viaducts

The static and dynamic behaviour of the viaducts was analysed by means of F.E. models including three spans connected by appropriate boundary conditions to represent the viaducts as if they had an indefinite number of spans, as shown in Figures 2.a and 2.b for the roadway and railway viaducts, respectively.



**Figure 2a.** General view of the numerical model of roadway viaduct

**Figure 2b.** General view of the numerical model of railway viaduct

### 3.2 Model of analysis of roadway traffic

The derivation of the traffic action in the most critical cross-sections of the roadway viaducts is based on a process of traffic flow simulation over typical viaducts, taking into account the different variables and their variability that define the vehicle configuration (axle loads, axle spacing) and the characteristics of the available traffic data recorded on the bridge site (Average daily traffic (ADT), percentage of trucks, distribution of weekly average daily traffic to each day in the week, peak hour factor, probability of congested traffic, mean number of trucks per platoon, distribution of trucks between lanes, impact factor). The model of traffic flow simulation has been developed at the Technical University of Catalunya (UPC) and is fully described in [1, 2].

In this case 200 weeks of traffic are simulated with a simulation period of the one week, obtaining the corresponding histogram and the parameters (mean value and standard deviation) of the random variable “maximum traffic action (or maximum traffic load effects) in one week”. The statistical distribution for the maximum effect of traffic is assumed to be of the Gumbel (Maximum effects) type. To obtain reliable results of the effects representative for longer periods of time (50,75,100 years) it will be necessary to obtain a long set of simulated data. Because this is not available, the method used here to extrapolate for longer periods of time is based on the analysis of the upper tail of the distribution through the use of extreme order statistics (General Pareto Distribution (GPD) [1]).

The characteristic values are defined for a 1000-year period, i.e., only 10 % of the time during a period of time of 100 years the traffic effect is larger than the respective value. The mean values correspond to the variable “maximum effect for a 100 year period”. In Table 3 are shown representative values of the traffic effect on the lateral girders of the roadway viaduct. In this table is shown that the maximum bending moments at mid-span of lateral pre-cast main girders resulting from the application of the loads prescribed by the DNV Specifications for design of highway bridges (original design) was 24 % lower than the characteristic value of probabilistic analysis and the maximum shear force of the original design in the halving joint was 36 % lower than the probabilistic model. All these results include the application of the corresponding impact factors.

**Table 3.** Representative values of the traffic effect on the roadway viaduct

Cross section	Mean value	Characteristic value	Code of Argentina
Bending moment at mid-span of the lateral pre-cast girders (M1)	8624 kNm	9222 kNm	6987 kNm
Shear force at the halving joint of the lateral cantilever (V1)	970 kN	941 kN	598 kN

The simulation process of the roadway traffic led to a coefficient of variation (c.o.v.) of the maximum longitudinal bending moments of about 8 %, but taking into account the size of the observed sample with respect to the total volume of present traffic and the tendencies of the traffic into the future [1], the probabilistic evaluation was conducted using a c.o.v. with a minimum value

of 15 % and a maximum of 25 % for this load effect. For the maximum bending moment in the transverse direction due to traffic, the c.o.v. was assumed to range from a minimum of 5 % to a maximum of 15 % even though the simulation process gave a much lower value for the load effect.

For the maximum shear forces due to traffic the simulation with the traffic model gave a c.o.v. of less than 2 %; however, the probabilistic evaluation was performed assuming values of the c.o.v. ranging from 10 to 15 % taking into account the trends for the composition of traffic in the future.

In the foregoing safety analysis both the minimum and maximum coefficients of variation of the traffic load effects were considered in order to analyse the influence of the variability of the load effects due to traffic on the reliability index.

### 3.3 Model of analysis of railway traffic

The weight of the locomotives is modelled as deterministic with the axle loads and distance between axles of the typical locomotive that crosses the bridge. The weight of the wagons is probabilistically modelled. The wagon axle-load is normally distributed with a mean value equal to the maximum wagon weight minus two times the standard deviation and a coefficient of variation of 3 %. With this definition of load for the event “passage of one train”, and the use of the influence lines obtained of the numerical model of viaduct, the mean value of the variables “effect in a cross-section due to the passage of a single train” is obtained. The coefficient of variation of those variables corresponding to a single event is also assumed to be 3 % and they are normally distributed.

Because the effect of a single event is modelled as a normal random variable, the probabilistic definition of the maximum effect due to a number of events (train passages) within a defined period of time is of the Gumbel type. The corresponding Cumulative Probability Function (CPF) is of the form:

$$F_{\max}(s) = \exp [- \exp(- \alpha(s-u))] \tag{1}$$

Where  $\alpha$  and  $u$  are the parameters of the probability function that may be derived considering the number of train passages in a defined period of time and the fact that  $F_s(s)$ , corresponding to the passage of a single train, is normally distributed. In fact, it can be found that:

$$u = F_s^{-1}(1 - 1/(\lambda T)) \quad \text{y} \quad \alpha = f_s(u) \lambda T \tag{2}$$

With  $F_s$  being the CPF of  $S$  (single event),  $f_s$  the probability distribution function of  $S$ ,  $\lambda$  the number of trains per year (730) and  $T$  the considered period of time (in years). Once the parameters  $\alpha$  and  $u$  are calculated, the mean value of the maximum effect within a period of time and the characteristic value for a defined return period are easily obtained. In this analysis, the period of time is 100 years, and the characteristic train effect is determined corresponding to the 90 % percentile in the 100-year extreme distribution (corresponding return period 1000 years).

Table 4 shows the main results obtained in the critical cross-sections of the girders. The impact factor is not yet considered. In this table are shown that the bending moment at the girder mid-span of the original design was 38 % larger than the characteristic value resulting of the probabilistic analysis, and the shear in the halving joint of original design was 41 % larger than the probabilistic model.

**Table 4.** Representative values of the traffic effect on the railway viaduct

Cross section	Mean value	Characteristic value	Original design	Code of Argentina
	c.o.v. = 3 %			
Bending moment at mid-span of the pre-cast girder (M3)	8810 kNm	8908 kNm	12279 kNm	11064 kNm
Shear force at the halving joint of the cantilever (V4)	823 kN	833 kN	1176 kN	1058 kN

#### 4. Probabilistic evaluation

The safety evaluation of the viaducts performed was based on the theory of structural reliability where the variables involved are considered to be random. Safety is expressed in terms of the Reliability Index ( $\beta$ ), or alternatively through the Probability of Failure, instead of the Safety Factor typical of a deterministic evaluation. Load effects and strength variables used in the evaluation were taken in accordance to design values.

##### 4.1 Statistical definition of variables: geometry, strength and load effects

Inspections and surveys performed on the viaducts indicate that the geometry of the structural components is very close to those foreseen in the design, and is therefore treated as deterministic according to design values in this evaluation. The material properties used in the foregoing analysis are those defined in [3, 4, 5] and given in Table 5.

*Table 5. Statistical definition of the variables of the resistance material*

Variable	Nominal resistance (MPa)	$\lambda =$ Mean / Nominal	Coef. of variation c.o.v. (%)	Type of distribution
Concrete of the cantilevers	26	1.20	15	Normal
Concrete of the pre-cast girder	38	1.35	10	Normal
Reinforcing steel	440	1.12	12	Lognormal
Prestressing steel	1450	1.04	2.5	Normal

The statistical definition of gravity and permanent load effects are given in Table 6.

*Table 6. Statistical definition of the variables of the action effects*

Action effects	$\lambda =$ Mean / Nominal	Coef. of variation c.o.v. (%)	Type of distribution
Bending moment and shear due to self weight of the main girders	1.03	8	Normal
Bending moment and shear due to self weight of the slab and transversal beams	1.05	10	Normal
Bending moment and shear due to dead load	1.10	15	Normal

##### 4.2 Limit State Function, definition of variables and Reliability Index

The Limit State Function ( $G$ ) defines the boundary that separates the safe and unsafe domains. A positive value of  $G$  is associated with a safe realisation of the variables and the opposite for a negative value.  $G$  is a random variable, and the probability of  $G$  becoming negative defines the probability of failure of a section. The nominal values of the gravity and permanent load effects were calculated on the basis of the original design drawings except for the thickness of the wear layer of the roadway viaducts that was found to be 12 cm.

For assessment of the ultimate limit state in flexure, the limit state function  $G$  is defined as:

$$G = M_{ru} - (M_{pp1} + M_{pp2} + M_{cp} + M_{traffic}) \quad (3)$$

and for the ultimate limit state in shear:

$$G = V_{ru} - (V_{pp1} + V_{pp2} + V_{cp} + V_{traffic} - V_p) \quad (4)$$

where all variables are random. The type of statistical distribution and parameters considered for the bending moment and shear forces due to self-weight of the main spans and cantilevers ( $M_{pp1}, V_{pp1}$ ), weight of the transverse beams and deck slab ( $M_{pp2}, V_{pp2}$ ), permanent loads ( $M_{cp}, V_{cp}$ ) and load effects due to traffic loads ( $M_{traffic}, V_{traffic}$ ) are given in Tables 5, 6, 7 and 8 according to [3, 4].

The vertical component of the pre-stressing force ( $V_p$ ) due to the curvature of the post-tensioned tendons of the main girders was taken into account as a random variable with normal distribution and with a ratio  $\lambda$  of the mean value to the nominal design value equal to 1 and a c.o.v. of 5 %. These values are in accordance to measurements performed earlier in two post-tensioned bridges before they were demolished [5].

The mean value and the c.o.v. of the ultimate bending strength ( $M_{ru}$ ) and shear strength capacity ( $V_{ru}$ ) were obtained by a simulation algorithm that takes into account the random characteristics of the variables given in Table 5.

### 4.3 Assessment of the halving joints (sections V1 y V4)

To evaluate the level of safety of the halving joints a special mechanical model of compression struts and tension members in bending and shear was developed, including the effect of the sloping bottom surface of the cantilever. According to this model for strength of the joints the limit state function for shear may be written as:

$$G = [ (V_{pp1} + V_{pp2} + V_{cp} + V_{traffic}) / \text{tg } \alpha ] \text{tg } \gamma + V_{ru} - (V_{pp1} + V_{pp2} + V_{cp} + V_{traffic}) \quad (5)$$

In this expression  $\alpha$  is the slope angle of the compressed struts and  $\gamma$  is the slope angle of the bottom of the cantilever. The shear strength of the section  $V_{ru}$  equals the summation ( $\sum (T_i + F_i)$ ) of the contribution of the vertical stirrups ( $T_i$ ) and the vertical component of the ultimate strength of the inclined reinforcement bars ( $F_i$ ).  $G$  is assumed to have a log normal distribution with a coefficient of variation of 12 %. The ratio between the mean and nominal values is  $\lambda = 1.12$ . A sensitivity analysis was performed regarding variations of the slope of the critical slope of the compression strut; the critical value of this angle for the roadway viaducts was found to be 62°, while for the railway viaduct was 67°.

The limit state function adopted for assessment of the halving joint in bending according to the proposed mechanical model of compression struts and tension members can be expressed as:

$$G = M_{ru} - (V_{pp1} + V_{pp2} + V_{cp} + V_{traffic}) a \quad (6)$$

In this expression ( $a$ ) represents the eccentricity of the reaction of the simply supported spans with respect to the section considered, and the ultimate bending moment of the joint,  $M_{ru} = \sum (F_i \times d_i + T_i \times d_i + H_i \times d_i)$ , equals the summation of the contribution of the vertical stirrups ( $T_i$ ), inclined and horizontal steel bars ( $F_i$  and  $H_i$ , respectively) times their respective lever arms ( $d_i$ ). This variable is assumed to have a log normal distribution with  $\lambda = 1.12$  and a c.o.v. of 12 %. Figure 3 shows a mechanical model of the halving joints.

### 4.4 Results of the probabilistic evaluation

Tables 7 and 8 summarise the results of the foregoing probabilistic evaluation of the roadway and railway viaducts.

## 5. Comparison of the results of probabilistic and deterministic evaluation

Tables 9 and 10 present the values of the reliability indexes given by the probabilistic evaluation and the corresponding safety factors from the deterministic evaluation obtained as the ratio of the nominal load effects and the corresponding ultimate strength of the critical sections.

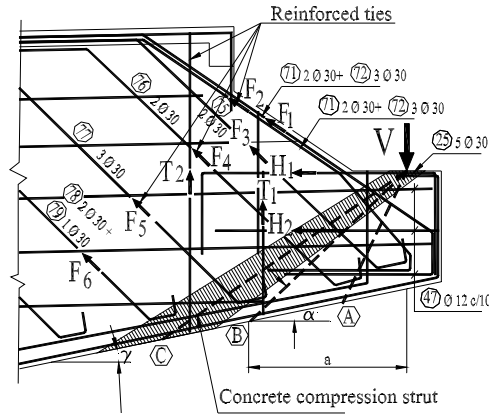


Figure 3. Mechanical model of the halving joints

Table 7. Summary of the probabilistic evaluation of the roadway viaducts. Reliability index  $\beta$ .

Section	Structural capacity			$S_{pp1}$	$S_{pp2}$	$S_{cp}$	$S_{traffic}$		$\beta$
	Mean value	$\lambda$	c.o.v. %				Mean value	c.o.v. %	
M1 [kNm]	30458	1.04	4	5174	1156	3391	8624	15 / 25	4.31 / 3.25
M2 [kNm]	36338	1.12	12	5243	1588	1617	3067	15 / 25	5.64 / 5.57
V1 [kN]	strut and tie model			500	167	333	941	10 / 15	3.84 / 3.45
V2 [kN]	8634	1.20	15	970	206	284	882	10 / 15	4.85 / 4.84
V3 [kN]	2950	1.12	12	372	88	255	823	10 / 15	6.19 / 5.56

$\lambda$  = mean value / nominal value; c.o.v. = coefficient of variation; M1 = bending moment of the pre-cast main lateral girders at mid-span; M2 = bending moment at the root of the lateral cantilevers; V1 = Shear force at the halving joints; V2 = Shear force at the root of the lateral cantilevers; V3 = Shear force at the support of the lateral pre-cast main girders.

Table 8. Summary of the probabilistic evaluation of the railway viaducts. Reliability index  $\beta$ .

Section	Structural capacity			$S_{pp1}$	$S_{pp2}$	$S_{cp}$	$S_{traffic}$		$\beta$
	Mean value	$\lambda$	c.o.v. %				Mean value	c.o.v. %	
M3 [kNm]	64504	1.04	4	11290	1029	4136	10976 / 11388 *	15	13.2 / 13.1
M4 [kNm]	77969	1.12	12	9183	1000	3116	8261 / 9045 **	15	6.00 / 5.92
V4 [kN]	Strut and tie model			1019	98	372	1019 / 1058 *	15	5.45 / 5.35
V5 [kN]	14622	1.20	15	1519	176	490	1274 / 1392 **	15	5.08 / 5.02
V6 [kN]	4038	1.12	12	813	78	304	902 / 941 *	15	6.37 / 6.29

$\lambda$  = mean value / nominal value; c.o.v. = coefficient of variation; \* impact factors 1.06/1.10; \*\* impact factors 1.01/1.10; M3 = bending moment of the pre-cast main girders at mid-span; M4 = bending moment at the root of the cantilevers; V4 = shear force at the halving joints; V5 = shear force at the root of the cantilevers; V6 = shear force at the support of the pre-cast main girders.

**Table 9.** Structural assessment of the roadway viaducts

Section	Reliability index $\beta$	Safety factor
M1	4.31	1.81
M2	5.64	2.55
V1	3.84	1.74
V2	4.85	2.97
V3	6.19	8.83

**Table 10.** Structural assessment of the railway viaducts

Section	Reliability index $\beta$	Safety factor
M3	13.2	2.23
M4	6.00	3.34
V4	5.45	1.67
V5	5.08	2.70
V6	6.37	3.72

## 6. Conclusions

The reliability indexes obtained were compatible with the minimum values stated in different National and International Codes accounting for ductile behaviour in conjunction with structural redundancy. A similar conclusion was arrived at with the deterministic approach incorporating field data such as measured dead weights and material strengths, together with the code-prescribed loads and using the numerical models calibrated with structural performance tests.

In fact the new AASHTO Code for design of bridges has been calibrated for a reliability index of 3.5 for the ultimate limit states and a lifespan of 75 years. On the other hand, the Euro code considers an objective value of 3.8 of reliability index for ultimate limit states and a lifespan of 100 years. It seems worthy to point out that for an existing structure lower values of the reliability index may be acceptable due to the reduction of the uncertainties of the variables that have been studied in detail at the site, such as an example, article 12 of the Canadian Code [6] allows a reduction of the objective reliability index of 0.50 for the case of gradual failures (non-brittle). Moreover, this same code allows an additional reduction of 0.25 for structural elements whose failure would not cause structural collapse due to structural redundancies. In this way the required reliability index reduces to  $2.75 = 3.5 - 0.5 - 0.25$  for the roadway viaducts due to its multiple girder structural configuration, and to  $3.0 = 3.5 - 0.5$  for the railway bridges without structural redundancy. All values given in Tables 6 and 7 are higher than these values accepted by this code.

The final conclusion is that both roadway and railway viaducts were found to have acceptable safety conditions according to the results of both probabilistic and deterministic methods of evaluation.

## 7. References

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